UNIT - 5
"GRAPH THEORY"

Date 11/01/23
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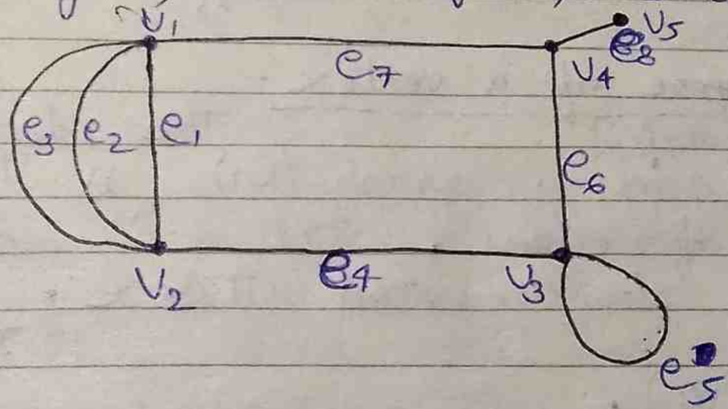
★ **Graph** :- A Graph $G = (V, E)$ consist of a set of object $V = \{V_1, V_2, V_3 ----\}$ whose elements are called vertices and a another ~~se~~ set $E = \{e_1, e_2, e_3, ---\}$ whose elements are called edges, and set of V, E is called a graph.

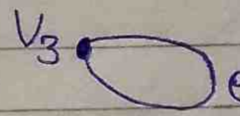
Example :-

Sari defination mai example likhna hai



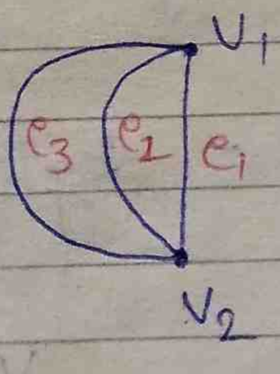
② **Self loop** :- (Self loop) An edge is said to be a self loop, if its both ~~and~~ end vertices are same.

example :-



③ **Parallel edge** :- If there are two or more than two having same pair of vertices, then such edges are called parallel edge

example :-

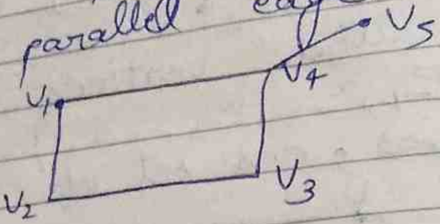


graph

PYQs

④ **Simple graph :-** A graph that has neither self loop nor parallel edge are called simple edge.

example :-



PYQs

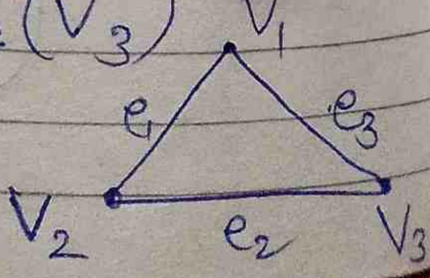
⑤ **Degree of a vertex :-** The degree of a vertex $deg(V_i)$ in a graph (G) is equal to the no. of edges, which are incident on V_i with self loop counted twice. It is denoted by $deg(V_i)$.

ex ⇒ $deg(V_1) = deg(V_2) = deg(V_3) = 4$

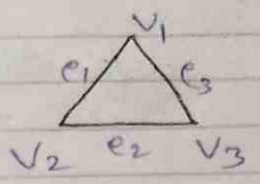
$deg(V_4) = 3$, $deg(V_5) = 1$

⑥ **Regular graph :-** A graph (G) in which all vertices are of equal degree is called regular graph.

Ex. ⇒ $deg(V_1) = deg(V_2) = deg(V_3)$

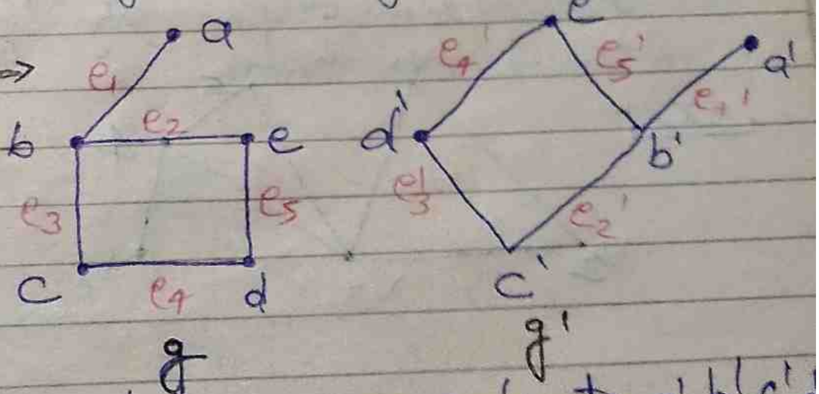


⑦ Isolated vertex :- A vertex of degree zero is called isolated vertex, or an end vertex.

ex. ⇒ 

• V_4 → V_4 is isolated vertex.

⑧ Iso-morphic graph :- Two graph are called iso-morphic, if there graph ~~theoretic~~ properties are same that is they have same no. of edge, they have equal no. of vertices with a given degree, they have same no. of vertices.

ex. ⇒ 

The vertices a, b, c, d, e correspond to $a', b', c', ($ respectively and the edges e_1, e_2, e_3, e_4, e_5 respective correspond to $e_1', e_2', e_3', e_4', e_5'$ respective

A graph g' is said to be sub-graph of graph, g. If all the vertices and all edges of 'G'' are in G.

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Each edge of (g') has the same end vertices in (g') as in g.

GATE → MCQs

Note :- From the defination of sub-graph

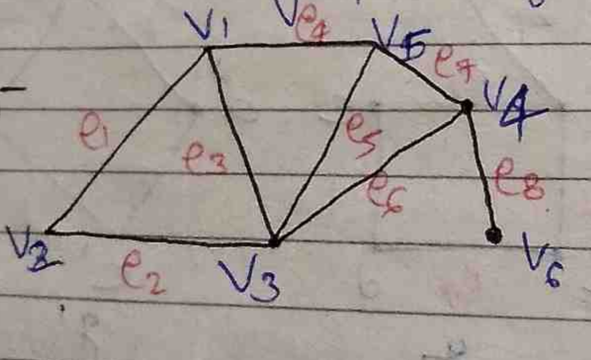
① Every graph is its own sub-graph.

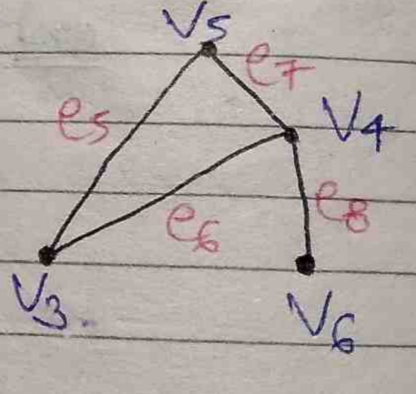
② A single vertex in (g) is a sub-graph of (g).

③ A single edge in (g) ~~is a~~ together with its end vertices is a sub-graph of (g).

④ A sub-graph of a sub-graph of (g) is also a sub-graph of (g).

ex. :-





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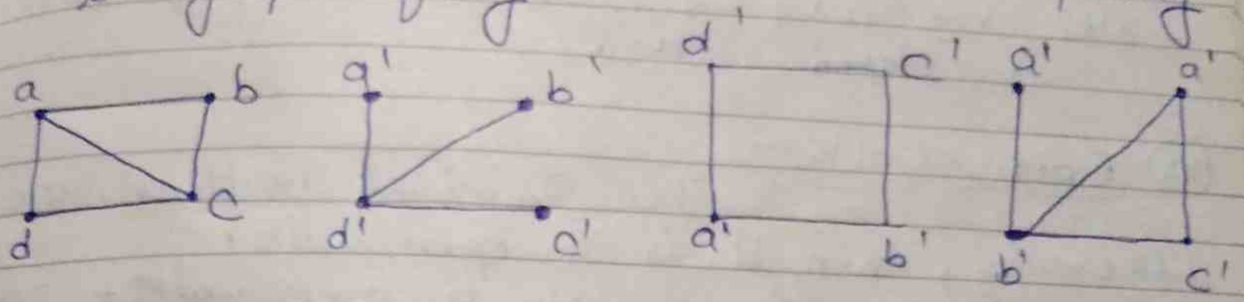
tices

⑨ Spanning sub-graph :-

Let $G' = (V', E')$ be subgraph of a graph $G = (V, E)$. If $V' = V$, then G' is said to be spaning sub-graph of g.



(g).

its

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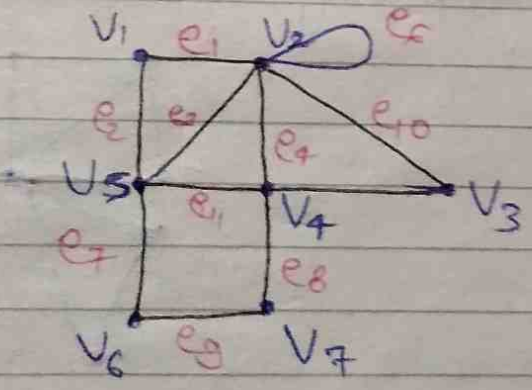
★⑩ Walk :-

A walk in a graph (g) is defined as a finite alternating sequence of vertices & edges which begins and ~~ends~~ ends, With vertices.



In a walk, no edge appear more than one however, a ~~verticle~~ vertex may appear more than once.

$\rightarrow V_1\, e_1\, V_2\, e_6\, V_2\, e_{10}\, V_3\, e_5\, V_4\, e_8\, V_7$

walk $\rightarrow$

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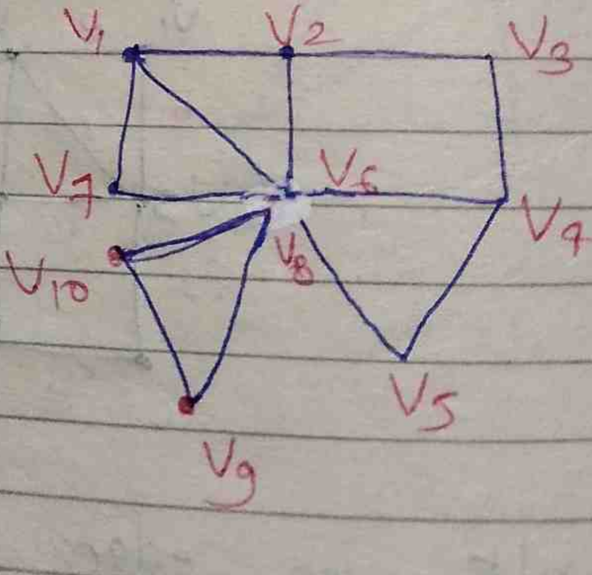
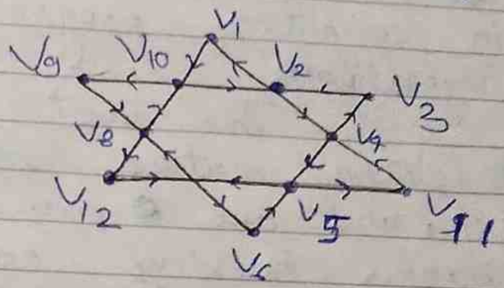
(11) **Closed Walk** :- If terminal vertices are same then walk is called closed walk. A walk $V_5 e_{11} V_4 e_5 V_2 e_6 V_2 e_1 V_1 e_2 V_5$ is a closed walk.

(12) **Open walk** :- When terminal vertices are difference, then it is a open walk. $V_5 e_1 V_4 e_5 V_7$ is a open walk.

(13) **Connected Graph** :- A graph (G) is called connected, if there is atleast one path b/w every pair of vertices. Otherwise ~~graph~~ (g) it is disconnected.



★ Euler graph :- A closed walk in a graph which include all the edges of the graph is euler graph.



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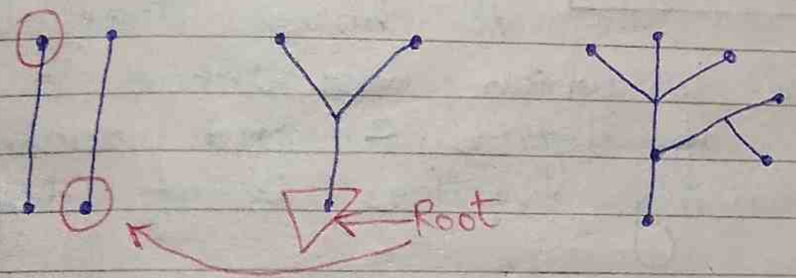
★ Tree :- A connected graph having no circuit is called tree.

TREE



• Some Types of Trees :-

① Rooted tree :- A rooted tree is a tree with distinguish vertices called the root.

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② ~~Decision~~ Decision tree :- A decision tree are labelled rooted tree which occur ~~labelled rooted tree~~ in application specially in computer programming & computer algorithm.

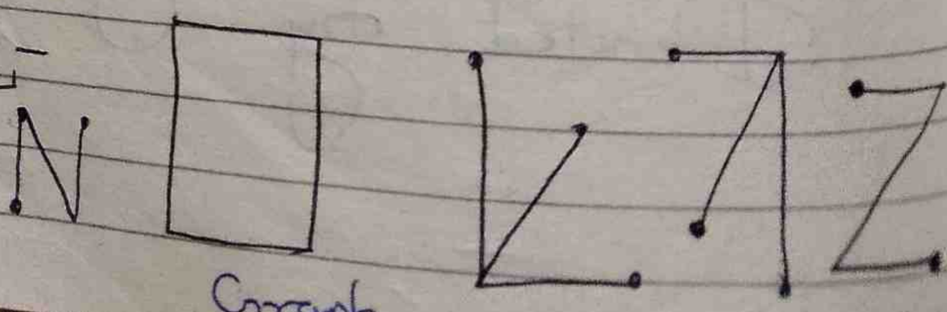
The root represent a starting point later vertices represent later decision point, and one proceed downward to the tree, chossing edge at each step occur to observed data.

③ Binary Tree :- A binary tree is defined as a tree in which ~~ver~~ there is exactly one vertex of degree 2 and each of the remaining vertices is of degree 1 or 3.

④ Spanning Tree :- If $g = (V, E)$ is any connected graph, a spanning tree in g is a sub-graph $T = (V, E')$ which is a tree.

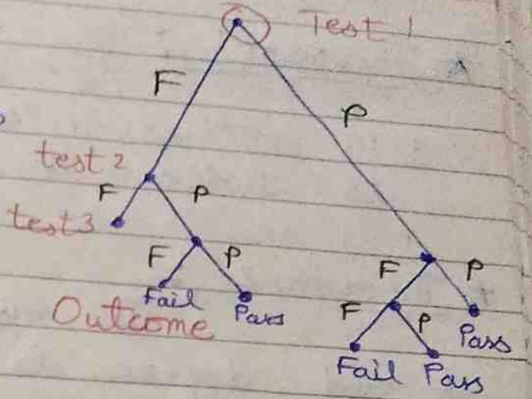
Example -

Spanning Tree



Graph

5. Binary decision tree:-



A student must pass two of three test to pass a course. The binary decision reappears in adjoining figure with F denoting Fail, P denoting pass.

If a student passes both of the first two test or fail both there is no decision, needed from test 3. We have ~~improvised~~ Emphasized the path to the outcome for a student who passes the first test fail the second and passes the third: the student passes the course.

In a binary tree, each decision has only two possible outcomes that is yes OR NO, True OR False, 0 OR 1).

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6-Marks Que

★ Theorem 1 :- The sum of the degrees of all vertices in a graph is equal to equal to twice the no. of edges.

★ Theorem 2 :- In any graph, the no. of vertices of odd degree is always even.

★ Theoram 5 :-

The maximum no. of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

★ Properties of double integration —

① If K is constant func.^{n} then, double integration are $\iint_R K f(x,y)\,dx\,dy = K\iint_R f(x,y)\,dx\,dy$

② Linear property of double integral

$$\iint_R \{K_1 f(x,y) + K_2 f(x,y)\}\,dx\,dy$$

$$= K_1\iint_R f_1(x,y)\,dx\,dy + K_2\iint_R f_2(x,y)\,dx\,dy$$

③ If the ~~reason are~~ region (R) is par into R_1, R_2

$$\iint_R f(x,y)\,dx\,dy = \iint_{R_1} f(x,y)\,dx\,dy) + \iint_{R_2} f(x,y)\,dx\,dy$$

④ The double integral $\int_a^b \int_{f_1(x)}^{f_2(x)} dx\,dy$ defines the area enclosed by the $y = f_1(x)$, $y = f_2(x)$ & the ordinates $x = a$, $x = b$.

(DNF) & (CNF)
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★ Involution law – $(a')' = a$

★ Demorgan's law – $(a+b)' = a'b'$
$(a.b)' = a' + b'$

★ In the Boolean Algebra, prove the following.

(i) $a' + ab = a' + b$
(ii) $a \cdot b \Rightarrow a \cdot b' = 0 \ \forall a, b \in B$
(iii) $a \cdot b + b \cdot c + c \cdot a = (a+b)(b+c)(c+a)$

★ Boolean func.^{n} –
An expression obtained by the application of binary operations '+' & '•' and a uniary operation (') on the finite number of elements of boolean-algebra (B, +, ') is called Boolean func.^{n} OR polynomial.

★ Minimal Boolean function –
A minimal boolean func.^{n} is (n) variable $x_1, x_2, x_3, \ldots x_n$ is product of (n) letters.

★ Disjunctive normal form (DNF) –
A boolen polynomial which can be written or sum of the minimal boolean function called disjuntive normal form (or conomial form)

$f(x,y) = x.y + x'y + xy' + x'y'$

2 mai comm...

★ **Conjuctive Normal form :-** A boolean polynomial is called a CNF (or dual canonical form) if it is the product of distinct factors where each factor is the sum of variables $x_1, x_2, x_3, \ldots$ and their products $x_1', x_2', x_3', \ldots$ and in each factor variables or their complements do not occur more than once

$$f(x_1, x_2) = (x_1 + x_2)(x_1' + x_2)(x_1 + x_2')(x_1' + x_2')$$

Ex.- Convert $x + x'y$ to DNF.

$$x(1) + x' \cdot y$$

$$x(y + y') + x'y$$

$$xy + xy' + x'y \quad \text{Ans.}$$

▸ Disjunctive (DNF)

Ex.- Write the following func.^{n} into conjuctive normal form, in which maximum number of variable are used.

(i) $f(x, y, z) = x \cdot y' + xz + xy$

Que Test for Consistency Solve :—
$\boxed{AX = B}$

$C = [A : B]$

$2x - 3y + 7z = 5$
$3x + y - 3z = 13$
$2x + 19y - 47z = 22$

Rank of A = Rank of C
then consistent

★ If Rank = no. of unknown (x, y, z)
then unique sol^n =>

★ Rank < no. of unknown => Infinite sol^n

$$AX = B$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 22 \end{bmatrix}$$

Now, Augumented Matrix

$$C = [A : B]$$

$$C = \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 22 \end{array}\right]_{3\times4}$$

For Finding Rank Convert in upper triangular Matrix

(—) Lower trianglular matrix

$R_2 = R_2 - R_2$ OR $2R_2 - 3R_1$
$R_3 = R_3 - R_1$

$$C = \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 22 & -54 & 17 \end{array}\right] \qquad R_3 = R_3 - 2R_2$$

27 ×2 40 17 (39) 22

$$C = \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

← Now it is upper-triangular matrix

A B

★ Rank of a matrix = No. of non-zero row is called rank of a matrix.

Now, In C matrix

No. of non-zero row = 3

$\boxed{\text{Rank of Matrix} = 3}$

Rank of a matrix find karne ka yeh rule hai Matrix Upper triangular matrix honi chahiye !!

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 0 & 11 & -27 \\ 0 & 0 & 0 \end{bmatrix}$$

Upper triang. matrix

Rank of a matrix = No. of Non-Zero Row

$\boxed{\text{Rank} = 2}$ of matrix A

[∴ Rank of C ≠ Rank of A]

[∴ The given system is not consistent and there exist No Sol.ⁿ]